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Analysis of the dynamic behavior of two parallel symmetric cracks using the non-local theory

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Abstract

In this paper, the dynamic behavior of two parallel symmetric cracks under harmonic anti-plane shear waves is studied using the non-local theory. For overcoming the mathematical difficulties, a one-dimensional non-local kernel is used instead of a two-dimensional one for the problem to obtain the stress occurs near the crack tips. The Fourier transform is applied and a mixed boundary value problem is formulated. Then a set of dual integral equations is solved using the Schmidt method. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip. The non-local elastic solutions yield a finite hoop stress at the crack tip, thus allowing for a fracture criterion based on the maximum stress hypothesis. The finite hoop stress at the crack tip depends on the crack length, the lattice parameter and the distance between two parallel cracks, respectively. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: The non-local theory; Schmidt method; The dual-integral equations; Two parallel cracks; The scattering of waves

1. Introduction

The last four decades have witnessed the inauguration of a novel theory of material bodies, named the non-local mechanics. This was done primarily due to the efforts of Edelen [1], Eringen [2], Green and Rivlin [3]. According to the non-local theory, the stress at a point X in a body depends not only on the strain at point X but also on those at all other points of the body. This is different from the classical theory. For the classical theory, the stress at a point X in a body

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depends only on the strain at point X. However, the solution of the classical theory contains stress singularity. This is not reasonable according to the physical nature. In papers [4–7], the state of stress near the tip of a sharp line crack in an elastic plate subjected to uniform tension, in-plane shear and anti-plane shear are discussed. The field equations employed in the solutions of these problems are those of the theory of the non-local elasticity. The solutions gave finite stress at the crack tips, thus resolving a fundamental problem that has remained unsolved over half a century. This enabled us to employ the maximum-stress hypothesis to deal with fracture problem and the composite materials problem in a natural way. Recently, the same problems in the papers [4–7] have been resolved in papers [8–10] by using the Schmidt method [11] and the results are more accurate and more reasonable. In papers [12–15], the problems for a crack or two cracks were investigated by using the non-local theory. To the author's knowledge, analytical treatment of two parallel symmetric cracks dynamic problem by using the non-local theory has not been attempted.

For the above-mentioned reasons, the present paper deals with the dynamic problem of two parallel symmetric cracks under harmonic anti-plane shear wave in an elastic plate by using the non-local theory. The field equations of non-local elasticity theory were employed to formulate and solve this problem. For overcoming the mathematical difficulties, one has to accept some assumptions as in Nowinski's works [16,17], one-dimensional non-local kernel function is used to instead of two-dimensional kernel function for the anti-plane problem to obtain the stress occur at the crack tips. Certainly, the assumption should be further investigated to satisfy the realistic condition. The Fourier transform is applied and a mixed boundary value problem is formulated. Then a set of dual integral equations is solved with the Schmidt method [11]. In solving the equations, the gaps of the displacement along the crack surface are expanded in a series of Jacobi polynomials. This process is quite different from that adopted in Eringen's works [4–7] and can overcome mathematical difficulty involved. The solution in this paper is accurate and reasonable. The solution, as expected, does not contain the dynamic stress singularity near the crack tips. The stress along the crack line depends not only on the crack length, the distance between two parallel cracks, but also on the lattice parameter. However, the stress resulting from the classical theory depends only on the crack length and the distance between two parallel cracks.

2. Basic equations of non-local elasticity

According to the non-local theory, the stress at a point X in a body depends not only on the strain at point X but also on those at all other points of the body. This observation is in accordance with atomic theory of the lattice and experimental observation of the phonon dispersion [18]. Basic equations of linear, homogeneous, isotropic, non-local elastic solids, with vanishing body force are:

$$\tau_{kl,k} = \rho \ddot{u}_l, \tag{1}$$

$$\tau_{kl} = \int_V \alpha(|X' - X|)\sigma_{kl}(X')\mathrm{d}V(X'), \tag{2}$$

where

$$\sigma_{ij}(X') = \lambda u_{r,r}(X')\delta_{ij} + \mu[u_{i,j}(X') + u_{j,i}(X')], \tag{3}$$

where the only difference from classical elasticity is in the stress constitutive equation (2) in which the stress $\tau_{kl}(X)$ at a point X depends on the strains $e_{kl}(X')$, at all points of the body. For homogeneous and isotropic solids there exist only two material constants, λ and μ are the Lame constants of classical elasticity. ρ is the density of the elastic materials. $\alpha(|X'-X|)$ is known as influence function, and is the function of the distance $|X'-X|$. The expression (3) is the classical Hook's law. Substitution of Eq. (3) into Eq. (2) and using Green–Gauss theorem, it can be obtained:

$$\int_V \alpha(|X'-X|)[(\lambda+\mu)u_{k,kl}(X',t) + \mu u_{l,kk}(X',t)]\mathrm{d}V(X') - \int_{\partial V} \alpha(|X'-X|)\sigma_{kl}(X',t)\mathrm{d}a_k(X') = 0. \tag{4}$$

Here the surface integral may be dropped if the only surface of the body is at infinity.

3. The crack model

It is assumed that there are two parallel symmetric cracks of length $2l$ in an elastic plate as shown in Fig. 1. h is the distance between two cracks. In this paper, the harmonic anti-plane waves are vertically incident. The z-axis is directed parallel to the crack's edges such that only non-vanishing displacement is the z-axis direction component, $w = w(x, y, t)$. Let ω be the circular frequency of the incident wave. $-\tau_0$ is a magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed but understood. As discussed in [4–7,19], when the crack is subjected to harmonic anti-plane shear waves, the boundary conditions on the crack faces at $y = 0$ are (in this paper, we just consider the perturbation stress field):

$$w^{(1)}(x,h,t) = w^{(2)}(x,h,t), \qquad \tau_{yz}^{(1)}(x,h,t) = \tau_{yz}^{(2)}(x,h,t), \quad |x| > l, \tag{5}$$

$$w^{(2)}(x,0,t) = w^{(3)}(x,0,t), \qquad \tau_{yz}^{(2)}(x,0,t) = \tau_{yz}^{(3)}(x,0,t), \quad |x| > l, \tag{6}$$

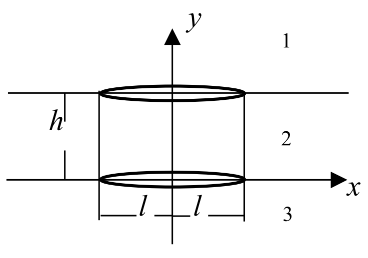

Fig. 1. Two parallel symmetric cracks in the plane.

$$\tau_{yz}^{(1)}(x,h,t) = \tau_{yz}^{(2)}(x,h,t) = -\tau_0, \quad |x| \leqslant l, \tag{7}$$

$$\tau_{yz}^{(2)}(x,0,t) = \tau_{yz}^{(3)}(x,0,t) = -\tau_0, \quad |x| \leqslant l, \tag{8}$$

$$w^{(1)}(x,y,t) = w^{(2)}(x,y,t) = w^{(3)}(x,y,t) = 0, \quad (x^2+y^2)^{1/2} \to \infty. \tag{9}$$

Note that all quantities with superscript k $(k = 1,2,3)$ refer to the upper half plane 1, the layer 2 and the lower half plane 3 as in Fig. 1, respectively. In this paper, we only consider that τ_0 is positive.

4. The dual integral equations

According to the boundary conditions, Eq. (4) can be written as follows:

$$\begin{aligned}&\mu\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\alpha(|x'-x|,|y'-y|)\nabla^2 w(x',y',t)\mathrm{d}x'\,\mathrm{d}y' - \int_{-l}^{l}\alpha(|x'-x|,0)[\![\sigma_{yz}(x',0,t)]\!]\mathrm{d}x' \\ &\quad - \int_{-l}^{l}\alpha(|x'-x|,h)[\![\sigma_{yz}(x',h,t)]\!]\mathrm{d}x' = -\rho\omega^2 w,\end{aligned} \tag{10}$$

where $[\![\sigma_{yz}(x,y,t)]\!] = \sigma_{yz}(x,y^+,t) - \sigma_{yz}(x,y^-,t)$ is a jump across the crack.

From the works [5,7], it can be obtained:

$$[\![\sigma_{yz}(x,0,t)]\!] = [\![\sigma_{yz}(x,h,t)]\!] = 0 \quad \text{for all } x. \tag{11}$$

Define the Fourier transform by the equations

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x)\mathrm{e}^{-\mathrm{i}sx}\mathrm{d}x, \tag{12}$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(s)\mathrm{e}^{\mathrm{i}sx}\mathrm{d}s. \tag{13}$$

For solving the problem, the Fourier transform of Eq. (10) with respect x can be given as follows:

$$\mu\int_{-\infty}^{\infty}\bar{\alpha}(|s|,|y'-y|)\left[(-s^2)\bar{w} + \frac{\partial^2\bar{w}}{\partial y^2}\right]\mathrm{d}y' = -\rho\omega^2\bar{w}. \tag{14}$$

What now remains is to solve the function w by using Eq. (14) and the boundary conditions. It seems obvious that a rigorous solution of such a problem encounters serious if not unsurmountable mathematical difficulties, and one has to resort to an approximate procedure. In the given problem, according to the assumptions as in Nowinski's works [16,17], the non-local interaction in y direction can be ignored. It can be given as

$$\bar{\alpha}(|s|, |y' - y|) = \bar{\alpha}_0(s)\delta(y' - y). \tag{15}$$

As discussed in [7,16,17], it was taken

$$\alpha_0 = \chi_0 \exp[-(\beta/a)^2(x' - x)^2] \quad \text{with } \chi_0 = \frac{1}{\sqrt{\pi}}\beta/a, \tag{16}$$

where β is a constant (here $\beta = e_0\sqrt{\pi}/l, e_0$ is a constant appropriate to each material), a is the lattice parameter. So it can be obtained

$$\bar{\alpha}_0(s) = \exp\left[-\frac{(sa)^2}{(2\beta)^2}\right], \tag{17}$$

where $\bar{\alpha}_0(s) = 1$ for the limit $a \to 0$, so that Eq. (14) reverts to the well-known equation of the classical theory.

From (14), we can derive

$$\frac{\partial^2 \bar{w}}{\partial y^2} - \left(s^2 - \frac{\rho\omega^2}{\mu\bar{\alpha}_0(s)}\right)\bar{w} = 0, \tag{18}$$

whose solutions do not present difficulties, we have

$$\bar{w}^{(1)}(s, y) = A_1(s)e^{-\gamma y} \quad (y \geqslant h), \tag{19}$$

$$\bar{w}^{(2)}(s, y) = A_2(s)e^{-\gamma y} + B_2(s)e^{\gamma y} \quad (h \geqslant y \geqslant 0), \tag{20}$$

$$\bar{w}^{(3)}(s, y) = A_3(s)e^{\gamma y} \quad (y \leqslant 0), \tag{21}$$

where $\gamma^2 = s^2 - \omega^2/c^2\bar{\alpha}_0(s)$, $c^2 = \mu/\rho$, $A_1(s)$, $A_2(s)$, $B_2(s)$ and $A_3(s)$ are to be determined from the boundary conditions.

The stress field, according to (2) and (3), is given by:

$$\tau_{yz}^{(1)}(x, y, t) = -\frac{2\mu}{\pi}\int_0^\infty \bar{\alpha}_0(s)\gamma A_1(s)e^{-\gamma y}\cos(sx)\mathrm{d}s \quad (y \geqslant h), \tag{22}$$

$$\tau_{yz}^{(2)}(x, y, t) = -\frac{2\mu}{\pi}\int_0^\infty \bar{\alpha}_0(s)\gamma[A_2(s)e^{-\gamma y} - B_2(s)e^{\gamma y}]\cos(sx)\mathrm{d}s \quad (h \geqslant y \geqslant 0), \tag{23}$$

$$\tau_{yz}^{(3)}(x, y, t) = \frac{2\mu}{\pi}\int_0^\infty \bar{\alpha}_0(s)\gamma A_3(s)e^{\gamma y}\cos(sx)\mathrm{d}s \quad (y \leqslant 0). \tag{24}$$

For solving the problem, the gap functions of the crack surface displacements are defined as follows:

$$f_1(x) = w^{(1)}(x, h^+) - w^{(2)}(x, h^-), \tag{25}$$

$$f_2(x) = w^{(2)}(x, 0^+) - w^{(3)}(x, 0^-). \tag{26}$$

Substituting Eqs. (19)–(21) into Eqs. (25) and (26), and applying the Fourier transform, it can be obtained

$$\bar{f}_1(s) = [A_1(s) - A_2(s)]\mathrm{e}^{-\gamma h} - B_2(s)\mathrm{e}^{\gamma h}, \tag{27}$$

$$\bar{f}_2(s) = A_2(s) + B_2(s) - A_3(s). \tag{28}$$

Substituting Eqs. (22)–(24) into Eqs. (5)–(8), it can be obtained

$$[A_1(s) - A_2(s)]\mathrm{e}^{-2\gamma h} = -B_2(s), \tag{29}$$

$$A_2(s) - B_2(s) = -A_3(s). \tag{30}$$

By solving four equations (27)–(30) with four unknown functions $A_1(s)$, $A_2(s)$, $B_2(s)$ and $A_3(s)$ and applying the boundary conditions (5)–(8), it can be obtained:

$$\int_0^\infty \frac{1}{2}\exp\left(-\frac{a^2s^2}{4\beta^2}\right)\gamma[\bar{f}_1(s) + \exp(-\gamma h)\bar{f}_2(s)]\cos(sx)\mathrm{d}s = \frac{\pi\tau_0}{2\mu}, \quad |x| \leqslant l, \tag{31}$$

$$\int_0^\infty \frac{1}{2}\exp\left(-\frac{a^2s^2}{4\beta^2}\right)\gamma[\exp(-\gamma h)\bar{f}_1(s) + \bar{f}_2(s)]\cos(sx)\mathrm{d}s = \frac{\pi\tau_0}{2\mu}, \quad |x| \leqslant l, \tag{32}$$

$$\int_0^\infty \bar{f}_1(s)\cos(sx)\mathrm{d}s = 0, \quad |x| > l, \tag{33}$$

$$\int_0^\infty \bar{f}_2(s)\cos(sx)\mathrm{d}s = 0, \quad |x| > l. \tag{34}$$

From (31)–(34), it can be obtained

$$\bar{f}_1(s) = \bar{f}_2(s) \Rightarrow f_1(x) = f_2(x), \quad \tau_{yz}^{(1)}(x,h,t) = \tau_{yz}^{(2)}(x,h,t) = \tau_{yz}^{(2)}(x,0,t) = \tau_{yz}^{(3)}(x,0,t) = \tau_{yz}. \tag{35}$$

Here we just solve the dual integral equations (31) and (33). Since the only difference between the classical and the non-local equations is in the introduction of the function $\exp(-a^2s^2/4\beta^2)$, it is logical to utilize the classical solution to convert the system (31)–(34) to an integral equation of the second kind which is generally better behaved. For $a = 0$, then $\exp(-a^2s^2/4\beta^2) = 1$ and Eqs. (31)–(34) reduce to the dual integral equations for same problem in classical elasticity. To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$, the dual-integral equations (31)–(34) must be solved.

5. Solution of the dual integral equation

The dual integral equations can be considered to be a single integral equation of the first kind with a discontinuous kernel [4]. It is well-known in the literature that integral equations of the first kind are generally ill-posed in sense of Hadamard, i.e., small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. In this paper, the Schmidt method was used to overcome the difficulty. The gap functions of the crack surface displacement are represented by the following series:

$$f_1(x) = f_2(x) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{1/2} \quad \text{for} -l \leqslant x \leqslant l,\ \ y=0, \tag{36}$$

where a_n is unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial [20]. The Fourier transform of Eq. (36) are [21]

$$\bar{f}_1(s) = \sum_{n=1}^{\infty} a_n G_n \frac{1}{s} J_{2n-1}(sl), \quad G_n = 2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n-1/2)}{(2n-2)!}, \tag{37}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq. (37) into Eqs. (31)–(34), respectively, Eqs. (33) and (34) have been automatically satisfied, Eq. (31) reduces to the form for $-l < x < l$,

$$\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} \frac{\gamma}{2s} \exp\left(-\frac{a^2 s^2}{4\beta^2}\right)[\exp(-\gamma h)+1] J_{2n-1}(sl)\cos(sx)\mathrm{d}s = \frac{\pi\tau_0}{2\mu}. \tag{38}$$

For a large s, the integrands of Eq. (38) are almost decreases exponentially. So they can be evaluated numerically by Filon's method (see e.g., [22]). Eq. (38) can now be solved for the coefficients a_n by the Schmidt method [11]. For brevity, Eq. (38) can be rewritten as

$$\sum_{n=1}^{\infty} a_n E_n(x) = U(x), \quad -l < x < l, \tag{39}$$

where $E_n(x)$ and $U(x)$ are known functions and the coefficients a_n are to be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_{-l}^{l} P_m(x)P_n(x)\mathrm{d}x = N_n\delta_{mn}, \quad N_n = \int_{-l}^{l} P_n^2(x)\mathrm{d}x \tag{40}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=1}^{n} \frac{M_{in}}{M_{nn}} E_i(x), \tag{41}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{11}, d_{12}, d_{13}, \ldots, d_{1n} \\ d_{21}, d_{22}, d_{23}, \ldots, d_{2n} \\ d_{31}, d_{32}, d_{33}, \ldots, d_{3n} \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ d_{n1}, d_{n2}, d_{n3}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_{-l}^{l} E_i(x)E_j(x)\mathrm{d}x. \tag{42}$$

Using Eqs. (39)–(42), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \quad \text{with } q_j = \frac{1}{N_j}\int_{-l}^{l} U(x)P_j(x)\mathrm{d}x. \tag{43}$$

6. Numerical calculations and discussion

From the works [8–10,12,23–25], it can be seen that the Schmidt method is performed satisfactorily if the first 10 terms of infinite series to Eq. (39) are retained. The behavior of the maximum stress stays steady with increase number in terms in Eq. (39). Although we can determine the entire the stress field from the coefficients a_n, it is of importance in fracture mechanics to determine the perturbation stress τ_{yz} in the vicinity of the crack tips. τ_{yz} along the crack line can be expressed, respectively, as

$$\tau_{yz} = -\frac{\mu}{\pi}\sum_{n=1}^{\infty} a_n G_n \int_0^{\infty} [\exp(-\gamma h) + 1]\frac{\gamma}{s}\exp\left(-\frac{a^2 s^2}{4\beta^2}\right)J_{2n-1}(sl)\cos(sx)\mathrm{d}s. \tag{44}$$

For $a = 0$ at $x = l$, we have the classical stress singularity. However, so long as $a \neq 0$, the semi-infinite integration and the series in Eq. (44) are convergent for any variable x. Eq. (44) gave a finite stress all along $y = 0$, so there is no stress singularity at the crack tips. At $-l < x < l$, τ_{yz}/τ_0 is very close to unity, and for $x > l$, τ_{yz}/τ_0 possesses finite values diminishing from a maximum value at $x = l$ to zero at $x = \infty$. Since $a/(2\beta l) > 1/100$ represents a crack length of less than 100 atomic distances as stated by Eringen [6], and such submicroscopic sizes other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions at such small crack sizes. The lattice parameter and the wave velocity are just considered in this paper. The semi-infinite numerical integrals, which occur, are evaluated easily by Filon and Simpson methods because the rapid diminution of the integrands. The results are plotted in Figs. 2–13. The following observations can be made from results:

(i) The maximum stress does not occur at the crack tip, but slightly away from it, as shown in Figs. 11–13. This phenomenon has been thoroughly substantiated by Eringen [26]. The maximum stress is finite. The distance between the crack tip and the maximum stress point is very small. This distance depends on the lattice parameter, the crack length and the circular fre-

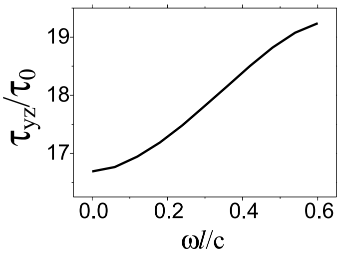

Fig. 2. The stress at the crack tip versus $\omega l/c$ for $h/l = 0.3$, $l = 1.0$, $a/2\beta l = 0.0005$.

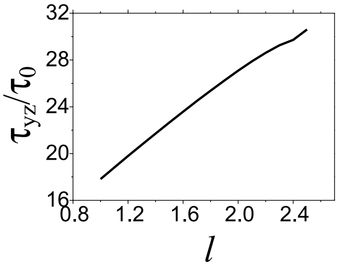

Fig. 3. The stress at the crack tip versus l for $\omega/c = 0.3$, $h = 0.3$, $a/2\beta = 0.0005$.

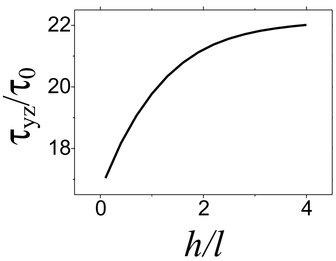

Fig. 4. The stress at the crack tip versus h/l for $\omega l/c = 0.3$, $l = 1.0$, $a/2\beta l = 0.0005$.

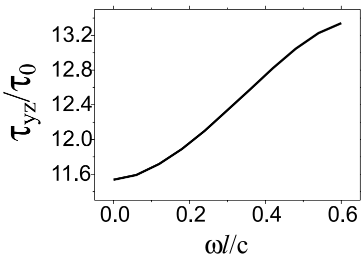

Fig. 5. The stress at the crack tip versus $\omega l/c$ for $h/l = 0.3$, $l = 1.0$, $a/2\beta l = 0.001$.

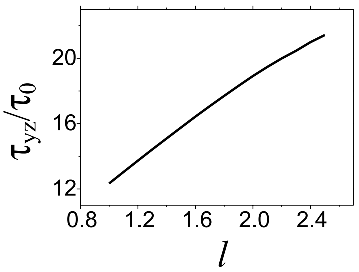

Fig. 6. The stress at the crack tip versus l for $\omega/c = 0.3$, $h = 0.3$, $a/2\beta = 0.001$.

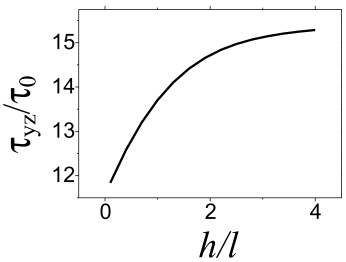

Fig. 7. The stress at the crack tip versus h/l for $\omega l/c = 0.3$, $l = 1.0$, $a/2\beta l = 0.001$.

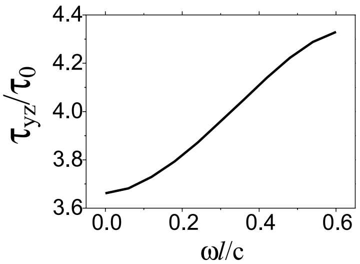

Fig. 8. The stress at the crack tip versus $\omega l/c$ for $h/l = 0.3$, $l = 1.0$, $a/2\beta l = 0.008$.

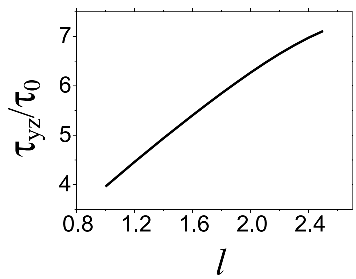

Fig. 9. The stress at the crack tip versus l for $\omega/c = 0.3$, $h = 0.3$, $a/2\beta = 0.008$.

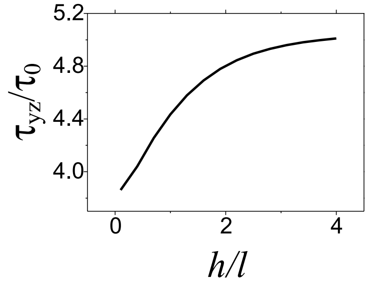

Fig. 10. The stress at the crack tip versus h/l for $\omega l/c = 0.3$, $l = 1.0$, $a/2\beta l = 0.008$.

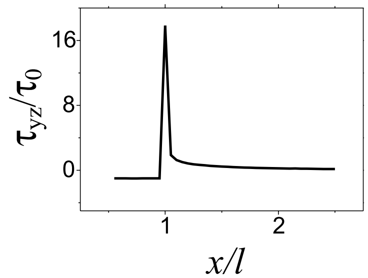

Fig. 11. The stress along the crack line versus x/l for $\omega l/c = 0.3$, $l = 1.0$, $h/l = 0.3$, $a/2\beta l = 0.0005$.

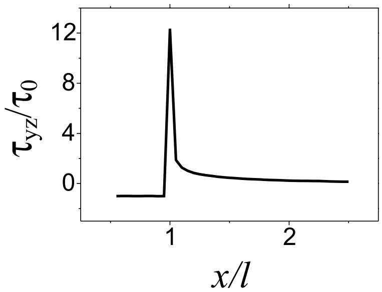

Fig. 12. The stress along the crack line versus x/l for $\omega l/c = 0.3$, $l = 1.0$, $h/l = 0.3$, $a/2\beta l = 0.001$.

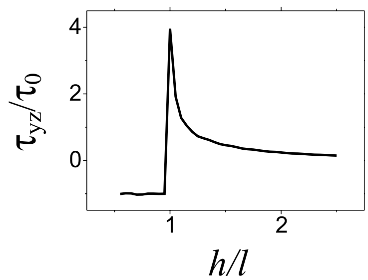

Fig. 13. The stress along the crack line versus x/l for $\omega l/c = 0.3$, $l = 1.0$, $h/l = 0.3$, $a/2\beta l = 0.008$.

quency of the incident wave. Contrary to the classical elasticity solution, it is found that no stress singularity is present at the crack tip, and also the present results converge to the classical ones for positions when far away from the crack tip.

(ii) The anti-plane shear stress at the crack tip becomes infinite as the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity.

(iii) The value of the stress concentrations (at the crack tip) increase with increase of the crack length, as shown in Figs. 3, 6 and 9. Noting this fact, experiments indicate that materials with smaller cracks are more resistant to fracture than those with larger cracks. However, the stress at the crack tip increases with increase of the distance between two parallel cracks. This phenomenon is called crack shielding effect, as shown in Figs. 4, 7 and 10.

(iv) The stress increases with increase of the frequency of the incident wave, as shown in Figs. 2, 5 and 8. The anti-plane shear stress at the crack tip increases with decrease of the lattice parameter.

(v) The significance of this result is that the fracture criteria are unified at both the macroscopic and microscopic scales.

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